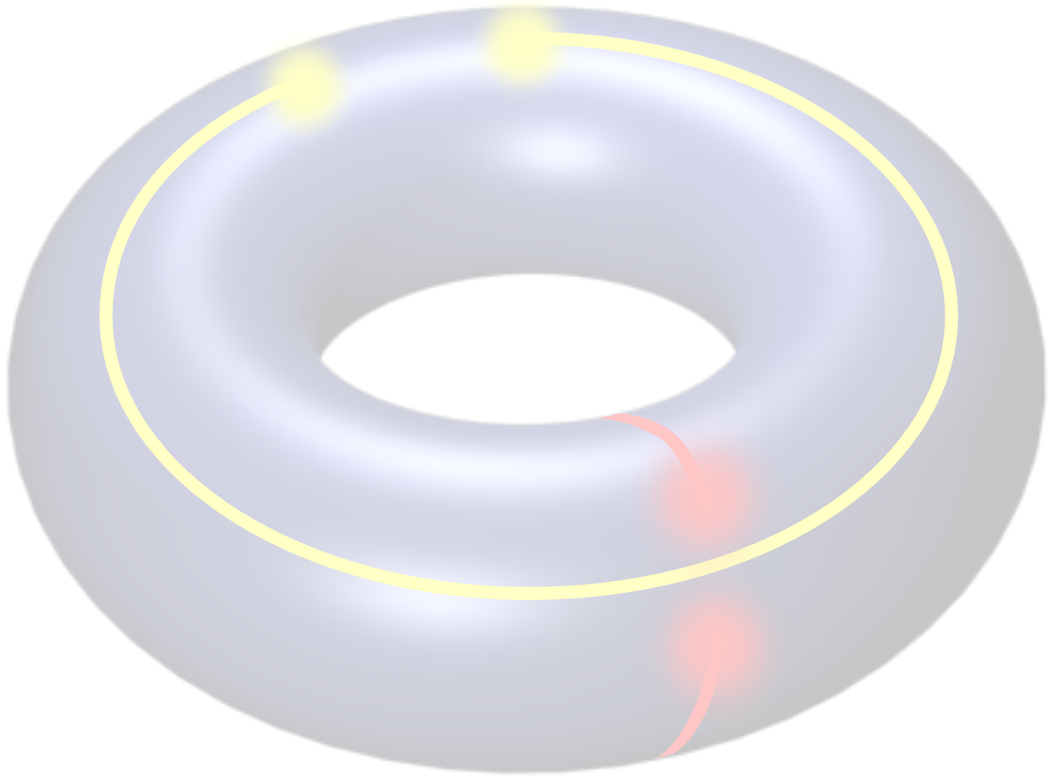
Detecting signatures of topological order from microscopic Hamiltonians

Frank Pollmann

Max Planck Institute for the Physics of Complex Systems



mpipks

FTPI, Minneapolis, May 2nd 2015

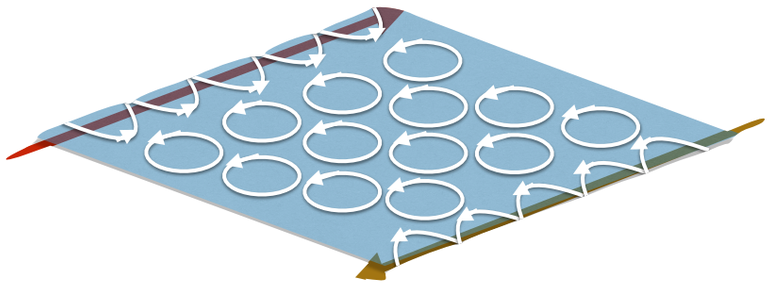
Detecting signatures of topological order from microscopic Hamiltonians

- Topological orders in "realistic" and correlated model Hamiltonians

  (i) Fractional quantum Hall effect
  [Tsui '82, Laughlin '83]

  (ii) Fractional Chern insulators
  [Neupert et al, Sun et al, Tang et al '11]

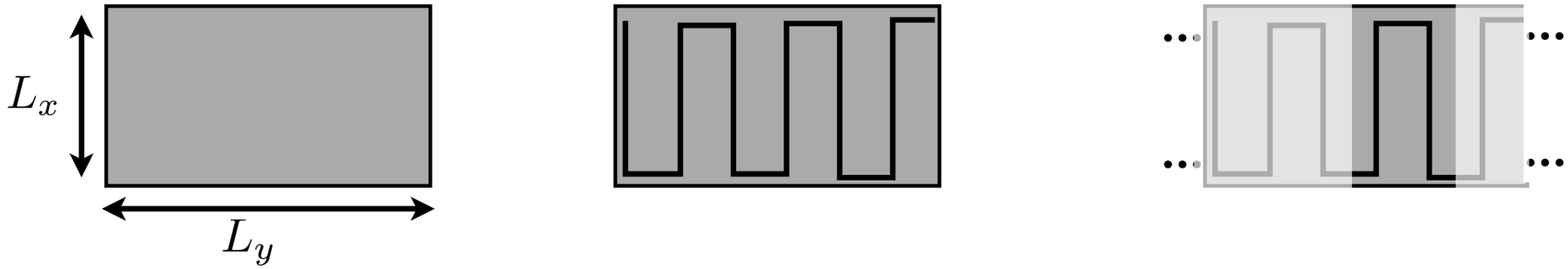
  (iii) Interaction driven Chern insulators
  [Raghu et al '08]

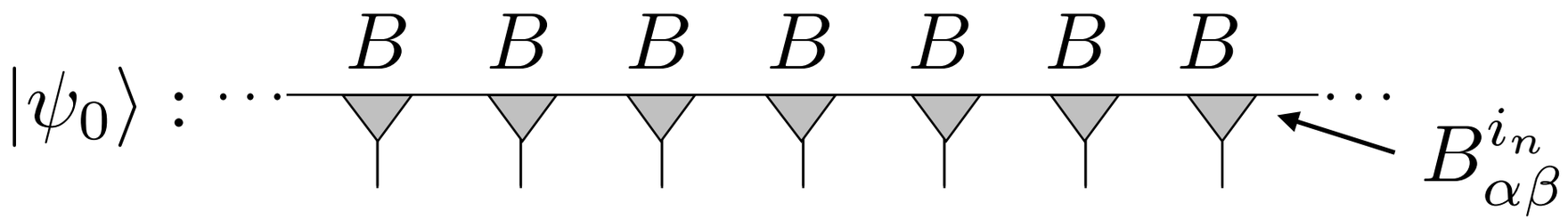


Detecting signatures of topological order from microscopic Hamiltonians

- Infinite Density Matrix Renormalization Group (iDMRG)
  [White '92, McCulloch '07]

ED: $\mathcal{O}(e^{L_x L_y})$ DMRG: $\mathcal{O}(L_y e^{L_x})$ iDMRG: $\mathcal{O}(e^{L_x})$

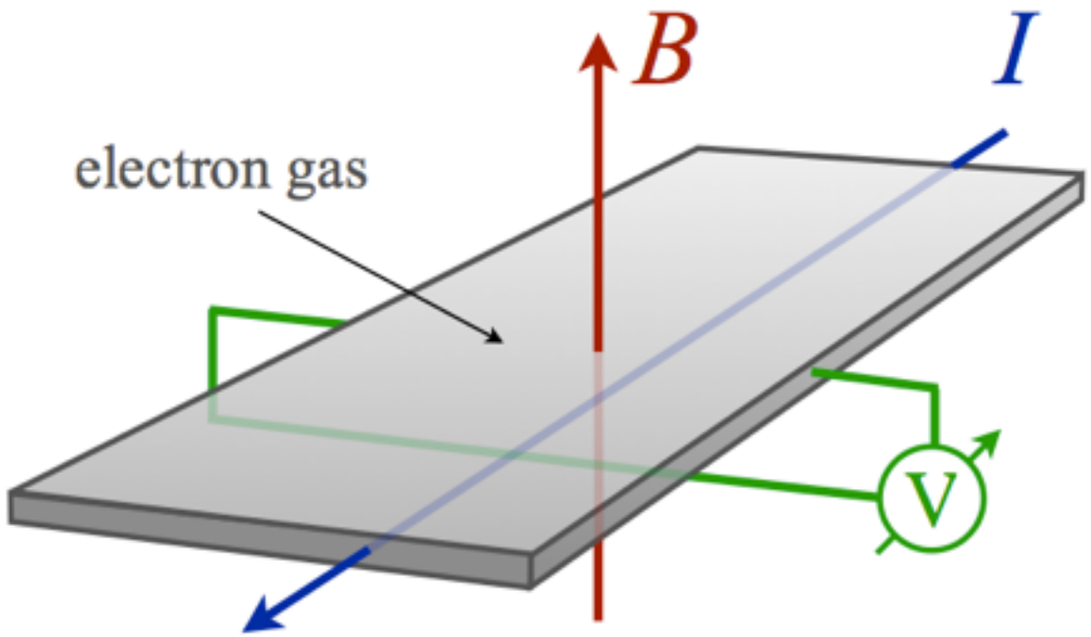




➡ Efficient variational calculation of the ground state

➡ Extract characteristic fingerprints of topological order

Fractional Quantum Hall

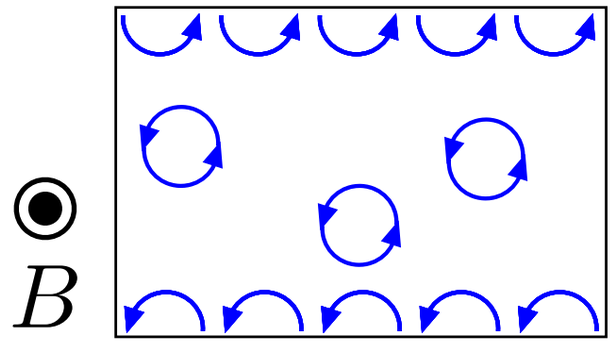


Mike Zaletel, Station Q
Roger Mong, Caltech
Zlatko Papic, Perimeter
Edward Rezayi, CSLA

Fractional Quantum Hall

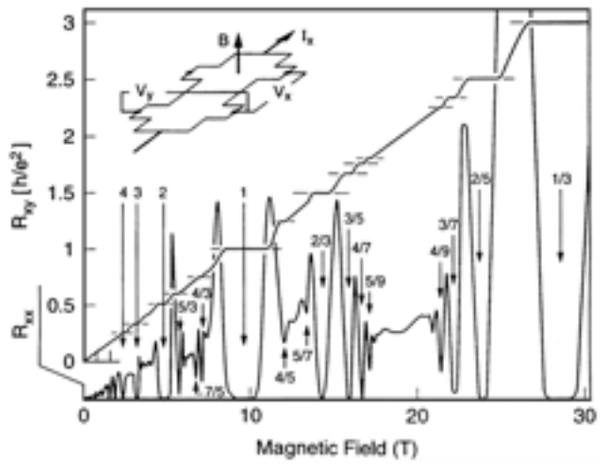
- 2D electron gas in magnetic field : B
  highly degenerate "Landau levels"

$$E_n = h\frac{eB}{m}(n+\frac{1}{2})$$

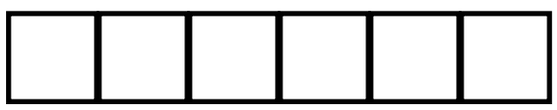


- Number N_B of degenerate orbits in each Landau level equal to number of flux quanta: Filling fraction $\nu = N_e/N_B$

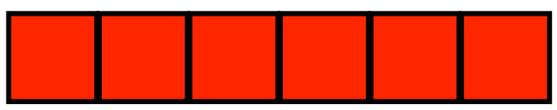
- Incompressible liquid at integer fillings
  [Klitzing '80]
- Fractional quantum Hall effect (FQHE):
  Incompressible liquid due to interactions
  [Tsui, Stormer '82, Laughlin '83]



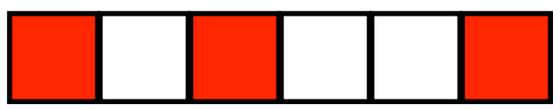
$n = 1$



N_B



Integer QH

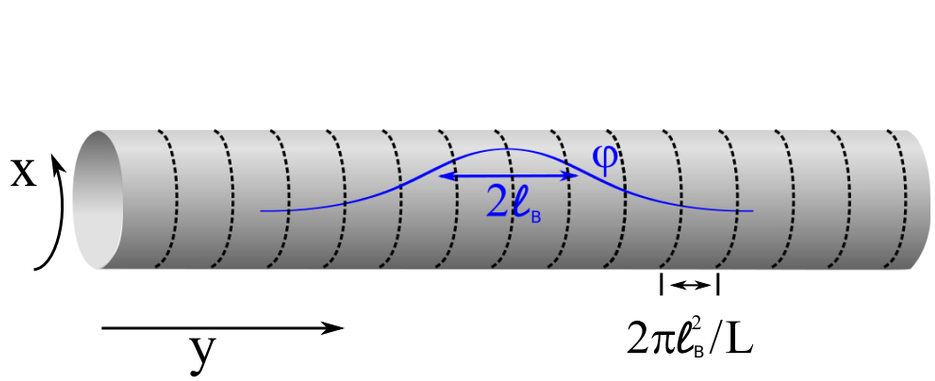


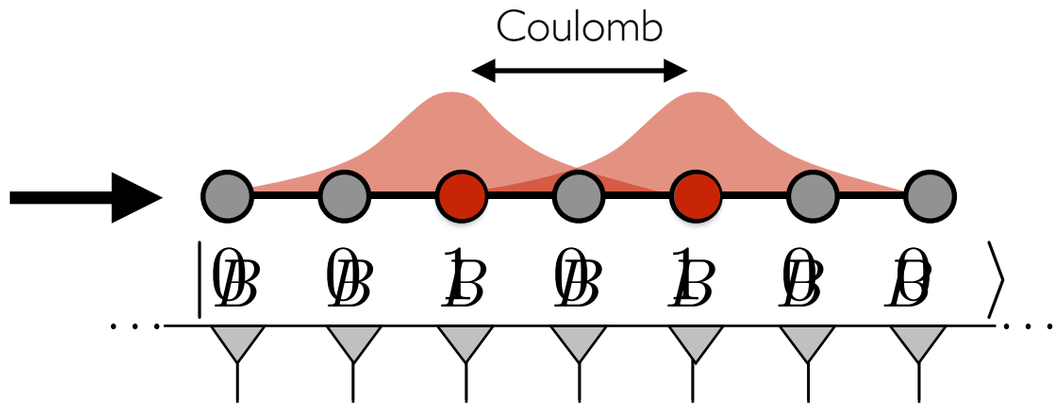
Fractional QH

Fractional Quantum Hall

- Consider the FQHE on an infinitely long cylinder
  - Orbitals are localized along the cylinder: Quasi 1D model using an occupation number basis $|\ldots, j_0, j_1, \ldots\rangle$

[Haldane & Rezayi '94; Bergholtz et al. '05, Seidel et al. '05]

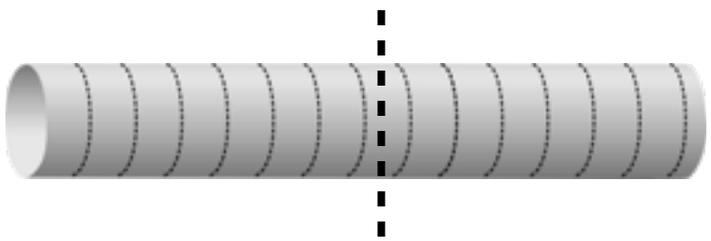




- Infinite DMRG allows for significantly larger system than accessible using exact diagonalization

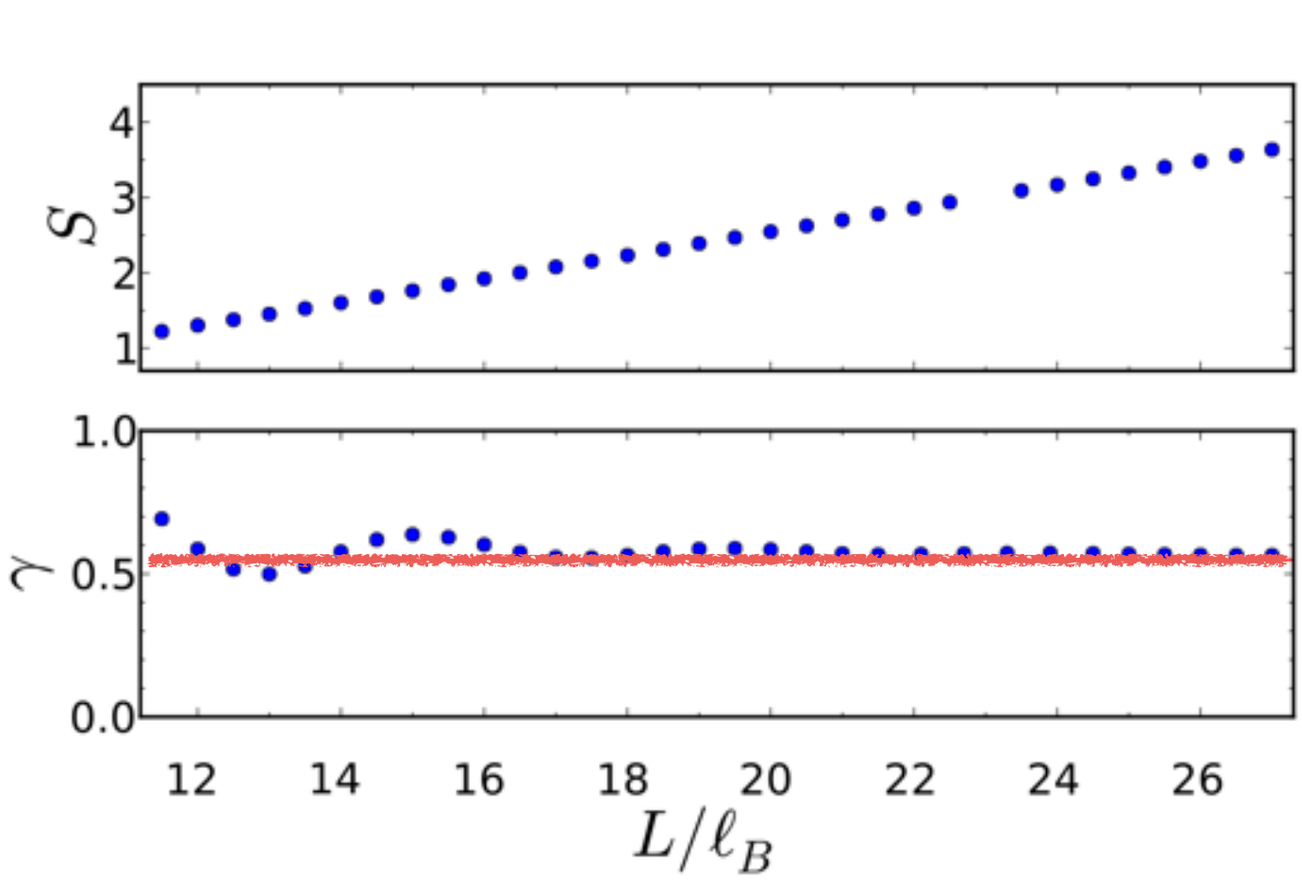
Fractional Quantum Hall

- Topological entanglement entropy of the $\nu = 1/3$ FQHE with Coulomb interactions



$$S = sL - \gamma_a$$

[Kitaev & Preskill, Levin & Wen '06]



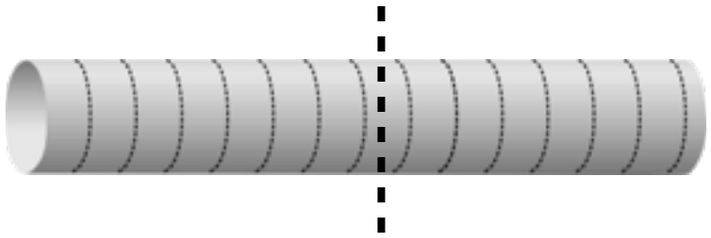
$\log \sqrt{3}$

M. P. Zaletel, R. S. K. Mong, FP, PRL 110, 236801 (2013).

M. P. Zaletel, Roger S. K. Mong, FP, and E. H. Rezayi, Phys. Rev. B 91, 045115 (2015).

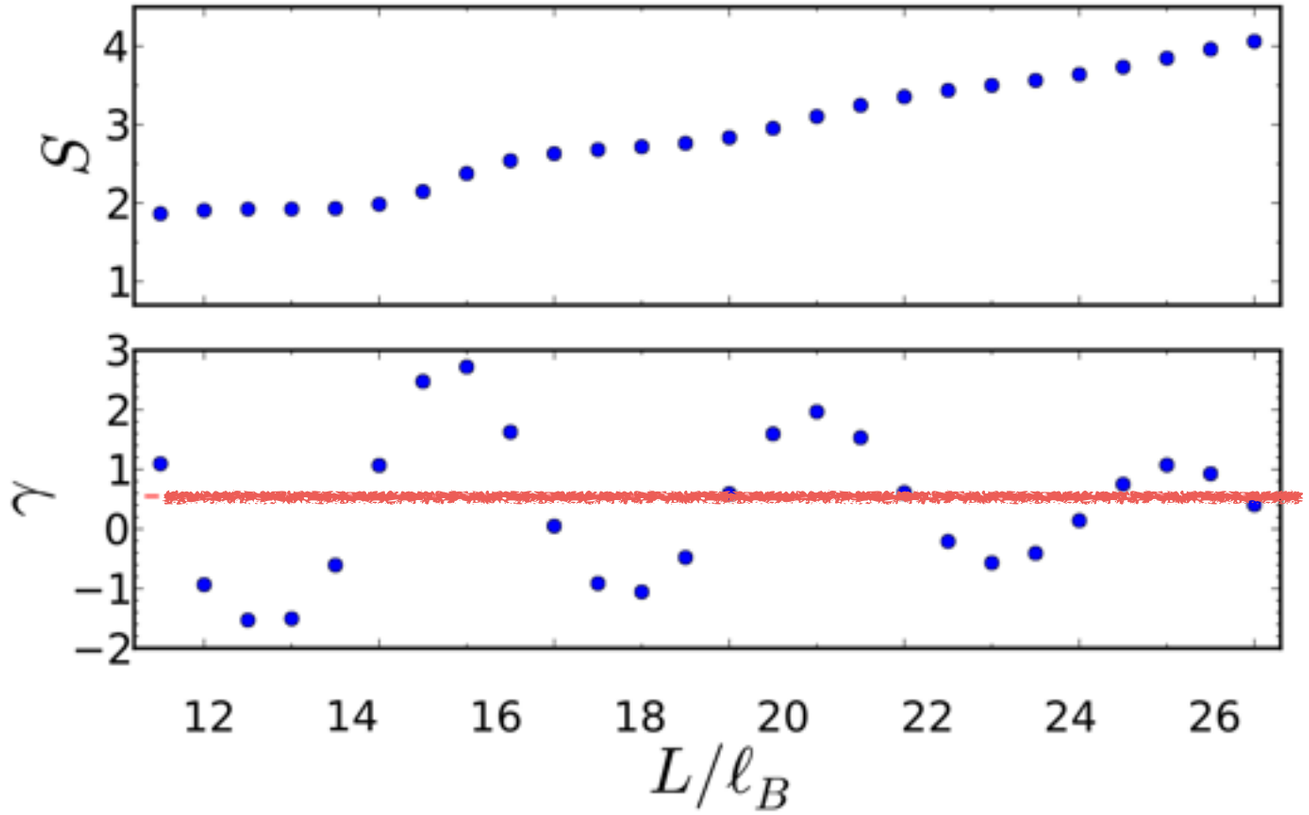
Fractional Quantum Hall

- Topological entanglement entropy of the $\nu = 7/3$ FQHE with Coulomb interactions



$$S = sL - \gamma_a$$

[Kitaev & Preskill, Levin & Wen '06]



$\log \sqrt{3}$

M. P. Zaletel, R. S. K. Mong, FP, PRL 110, 236801 (2013).

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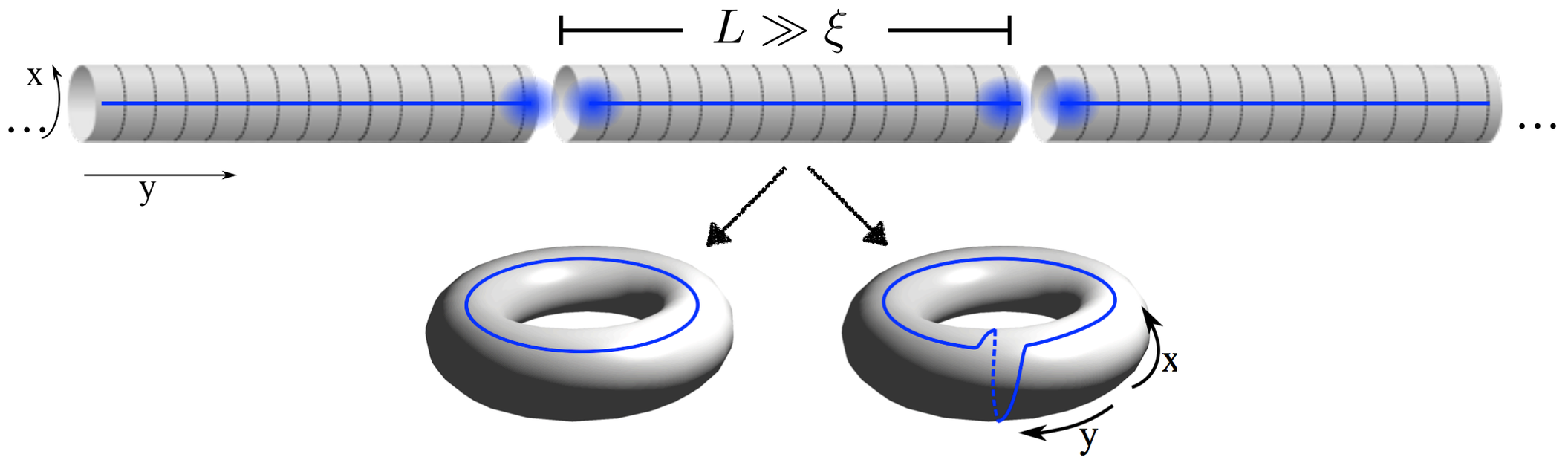
Fractional Quantum Hall

- Extracting topological content by adding a "twist"



- Momentum polarization: topological spin, central charge, Hall viscosity

$$U_{T;ab} = \delta_{ab} \exp\left[2\pi i \left(h_a - \frac{c}{24} - \frac{\eta_H}{2\pi\hbar} L_x^2\right)\right]$$

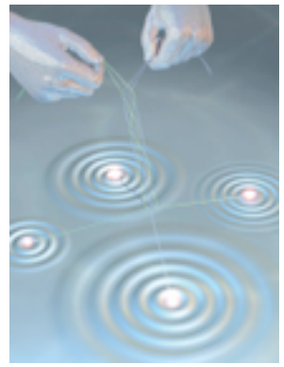
(see also Zhang et al. '12, Tu et al. '13, Cincio & Vidal '13)

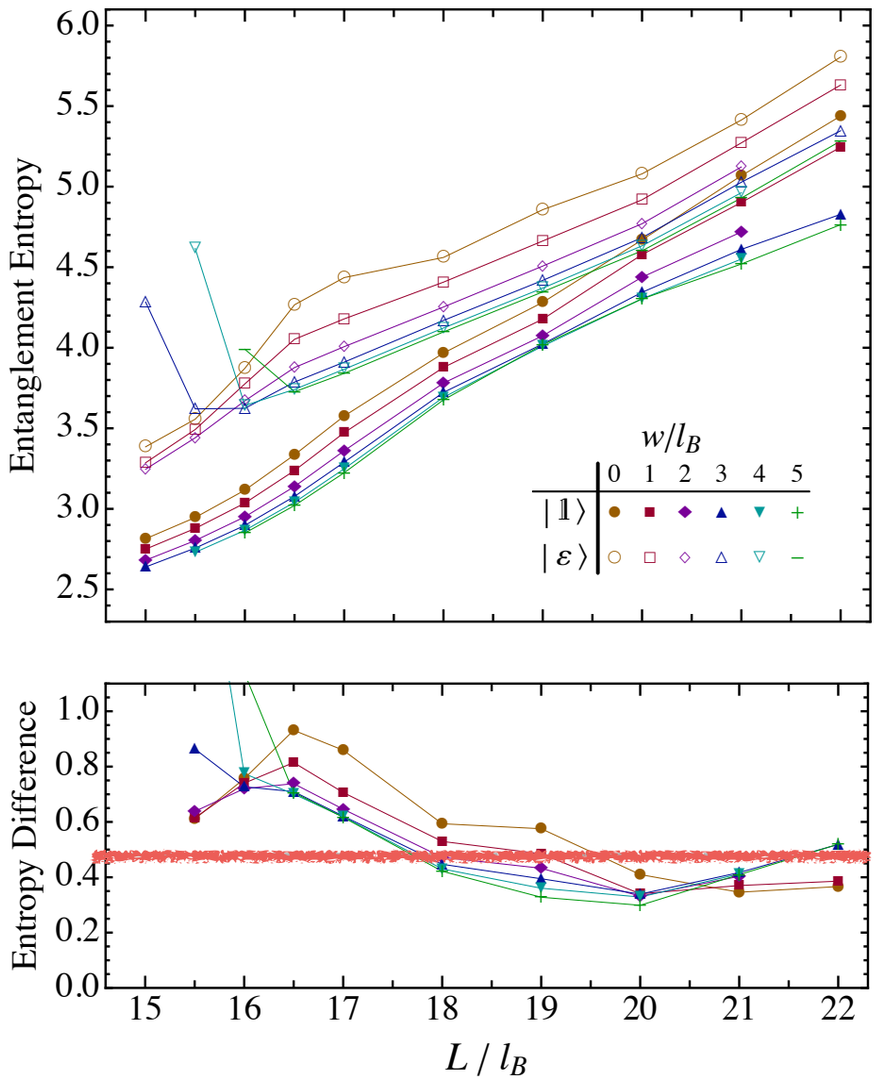
M. P. Zaletel, R. S. K. Mong, FP, J. Stat. Mech. P10007(2014).
M. P. Zaletel, R. S. K. Mong, FP, PRL 110, 236801 (2013).

Fractional Quantum Hall

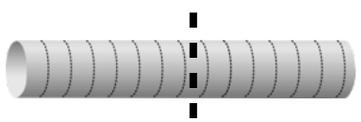
- iDMRG on FQHE at $\nu = 12/5$:
  Numerical evidence for the existence of Fibonacci anyons! [Read & Rezayi '98]

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.61803$$

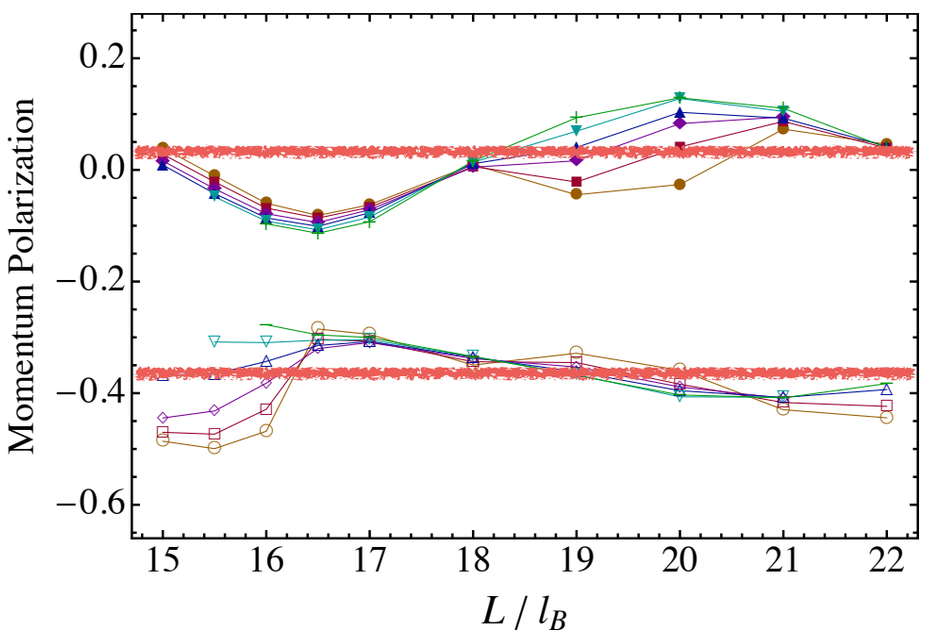


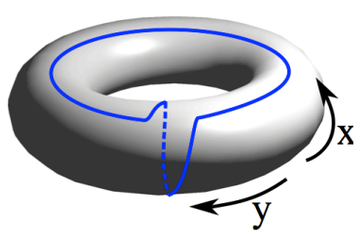


$$S = sL - \gamma_a \qquad \gamma_a = -\log\left(d_a / \sqrt{\sum_b d_b^2}\right)$$



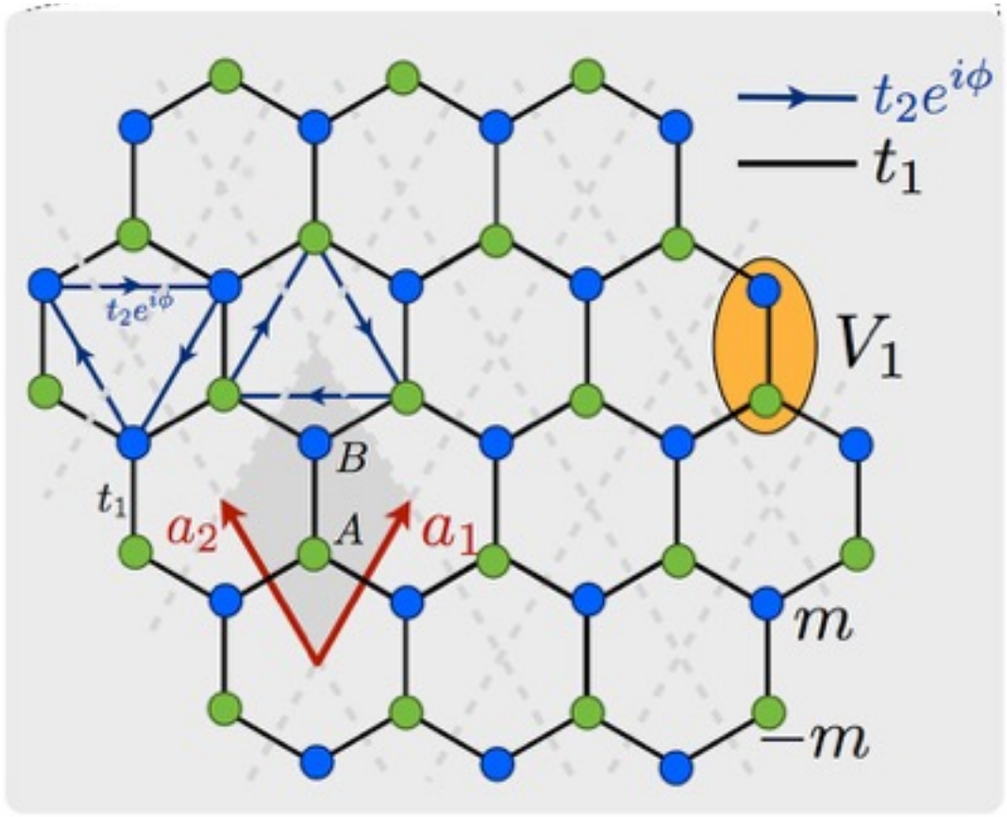
[Kitaev & Preskill, Levin & Wen '06]



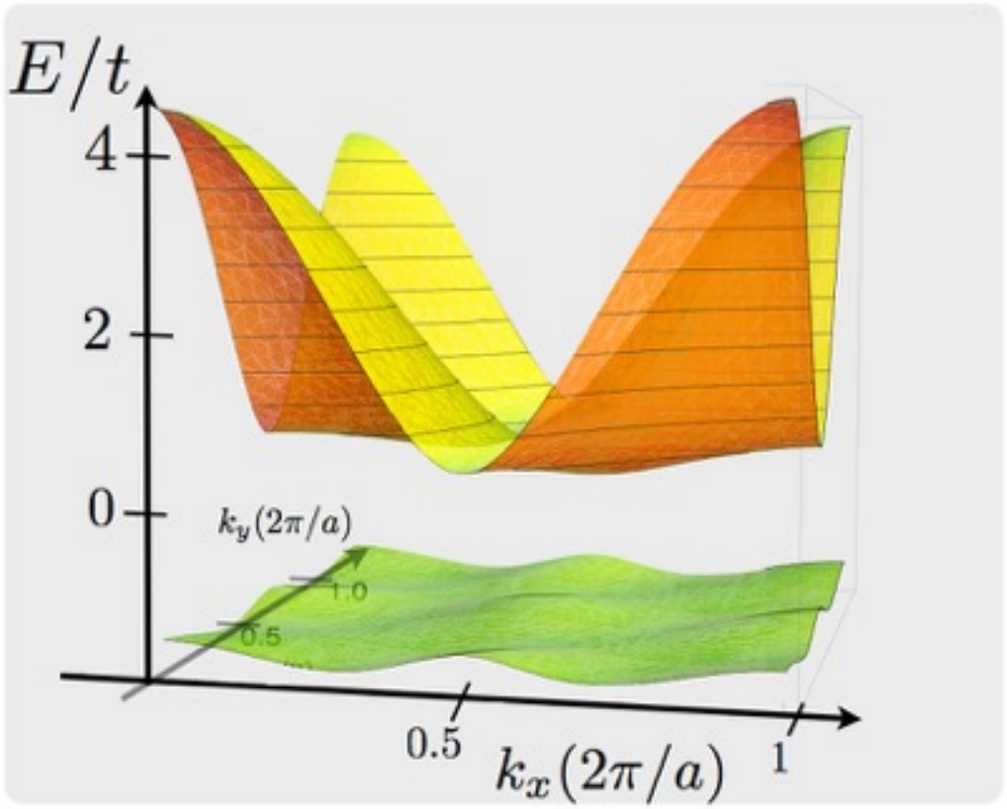


M. P. Zaletel, R. S. K. Mong, Z. Papic, FP, (in preparation).

Fractional Chern Insulators



[Haldane '88]



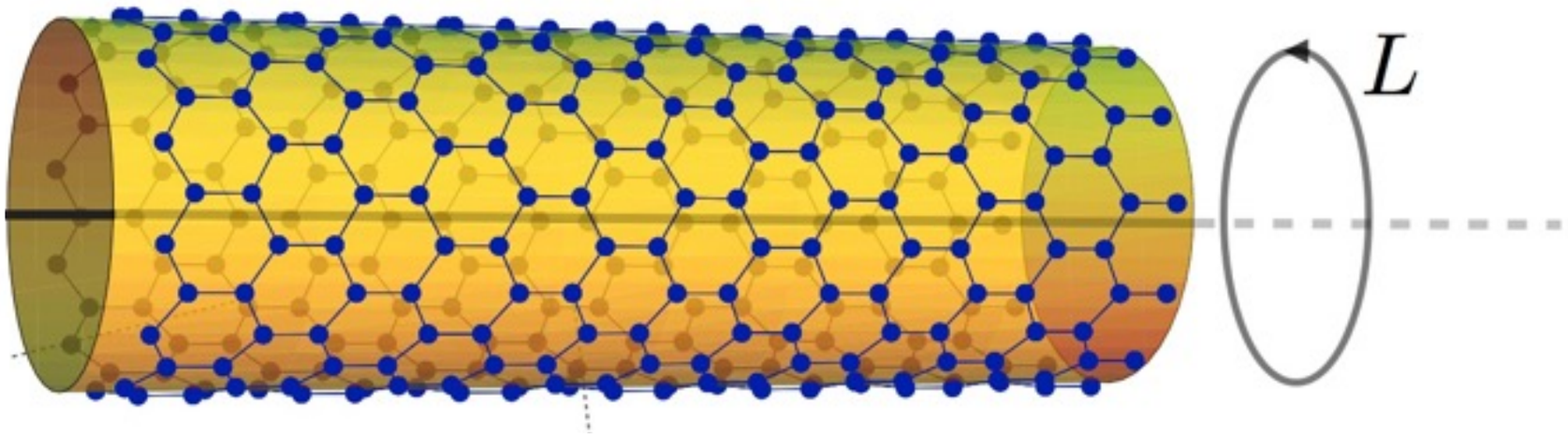
[Neupert et al, Sun et al, Tang et al '11]

D. Sheng, Z.-C. Gu, K. Sun, and L. Shen '11
N. Regnault and B. A. Bernevig '11
E. J. Bergholtz and Z. Liu '13
S. Kourtis, T. Neupert, C. Chamon, and C. Mudry, '12
S. Kourtis, J. W. F. Venderbos, and M. Daghofer '13
….

Fractional Chern Insulators

- Density Matrix Renormalization Group (DMRG) : Fractional Chern Insulators

  - $\nu = 1/3$ filling of the lowest band
  - Circumferences up to $L = 12$ sites

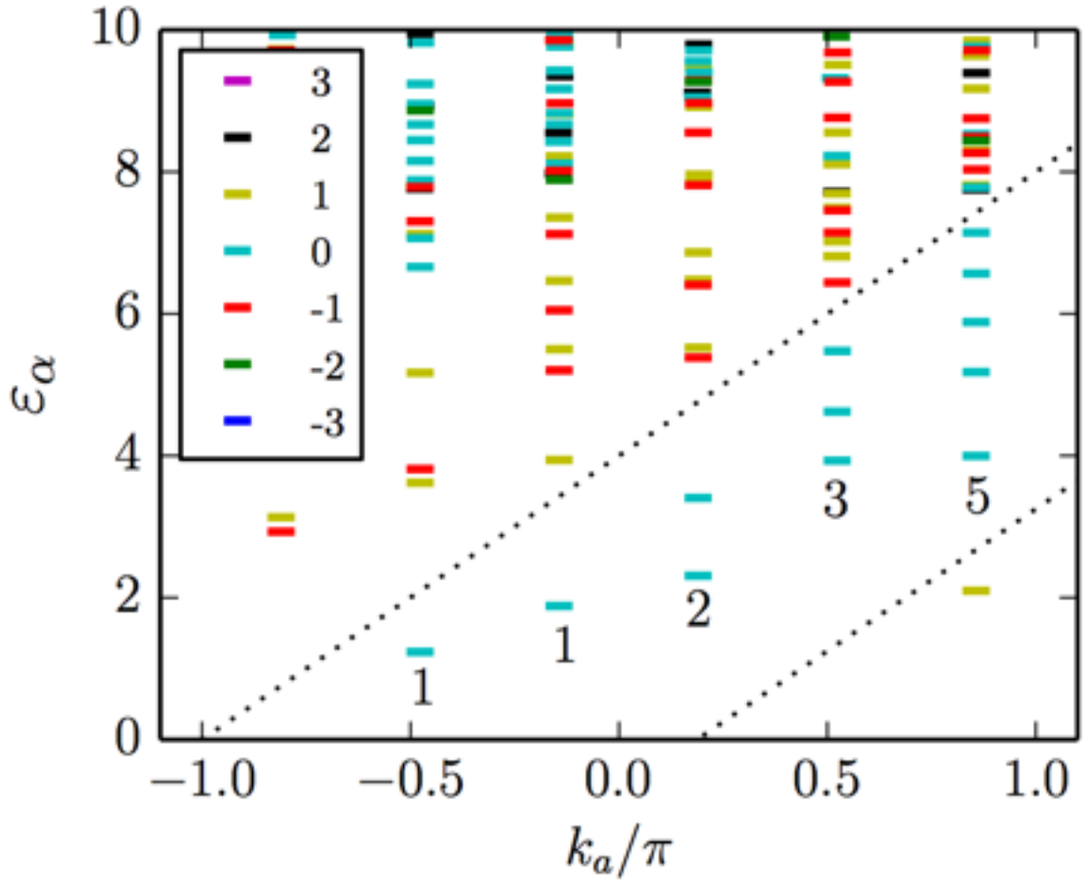


Adolfo Grushin, MPIPKS
Johannes Motruk, MPIPKS
Mike Zaletel, Station Q

A. Grushin, J. Motruk, M. P. Zaletel, FP, Phys. Rev. B 91, 035136 (2015).

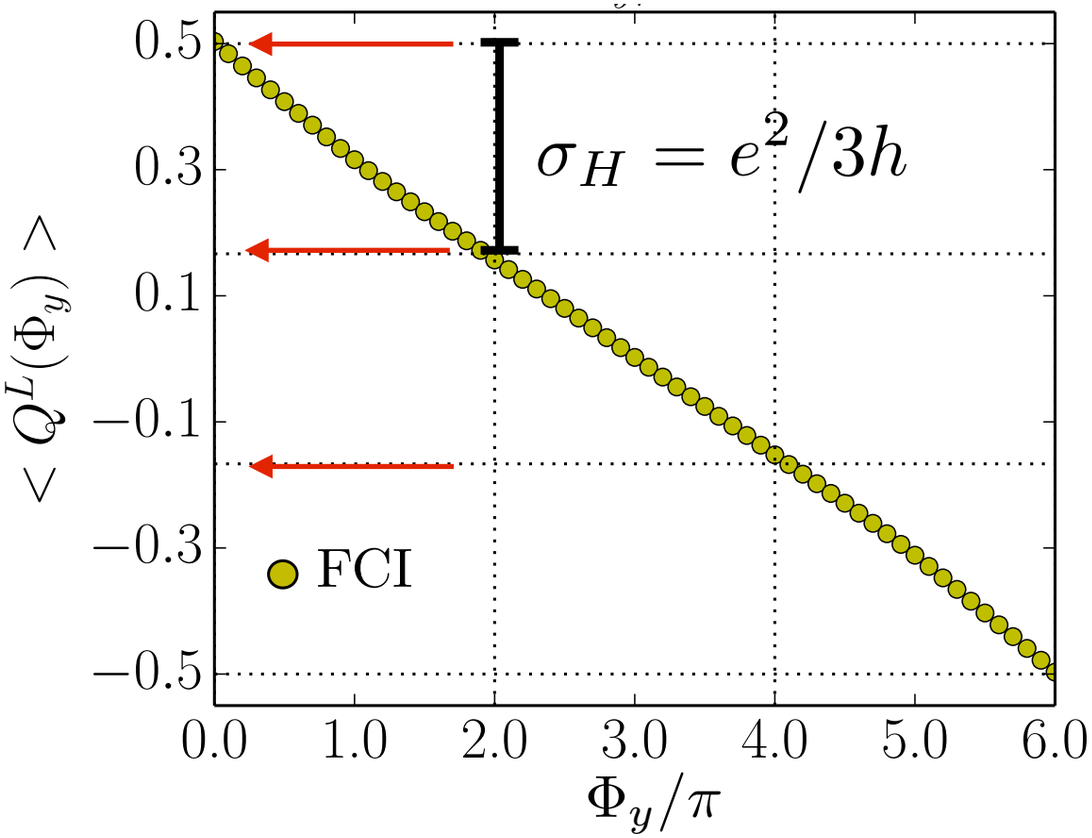
Fractional Chern Insulators

"CFT counting"



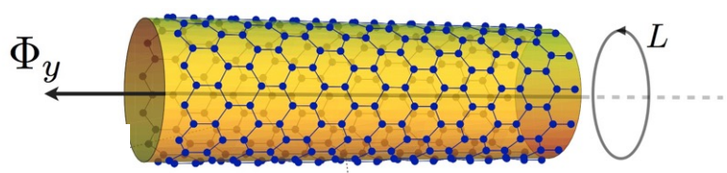
[Li & Haldane '08]

"Charge pumping"



[Laughlin '81]

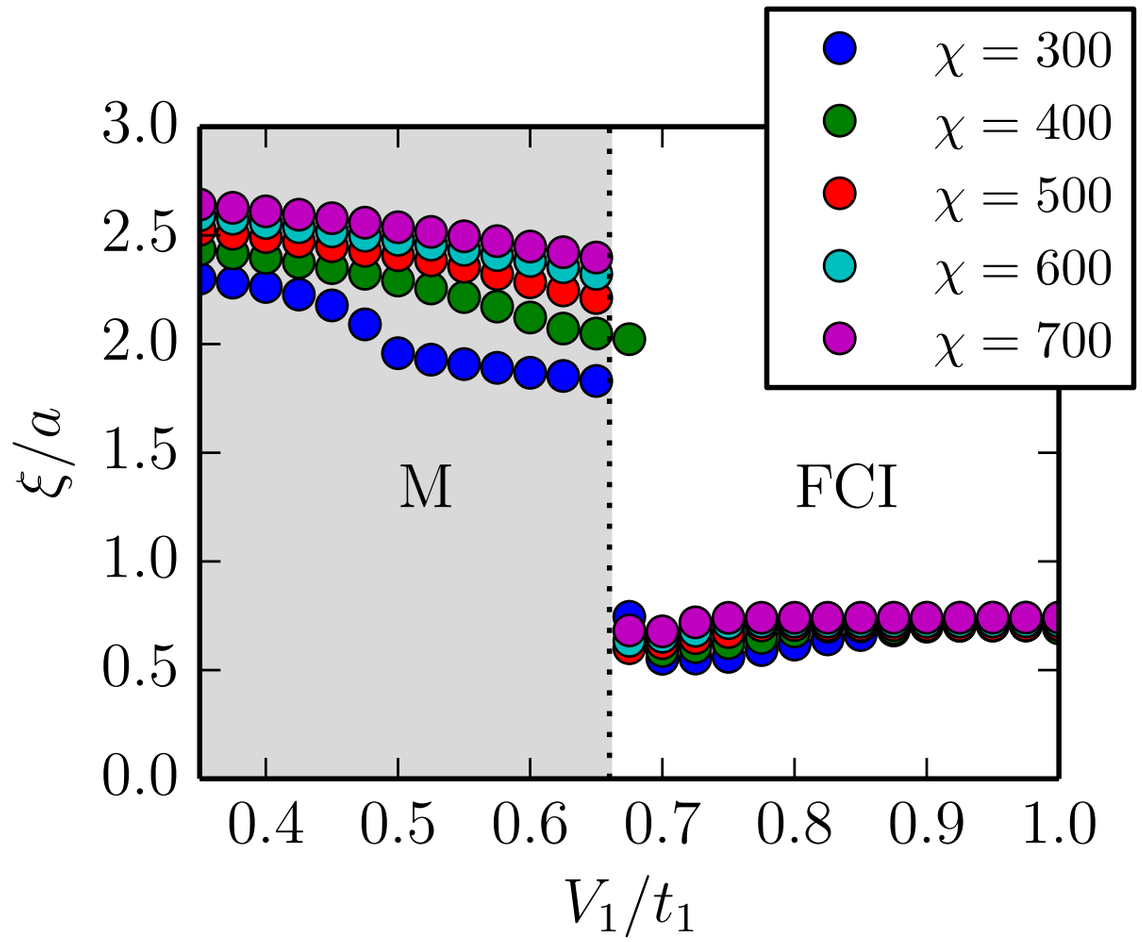
$$\rho^{\mathrm{red}} = \sum_\alpha \exp(-\epsilon_\alpha)|\phi_\alpha\rangle\langle\phi_\alpha|$$



A. Grushin, J. Motruk, M. P. Zaletel, FP, Phys. Rev. B 91, 035136 (2015).

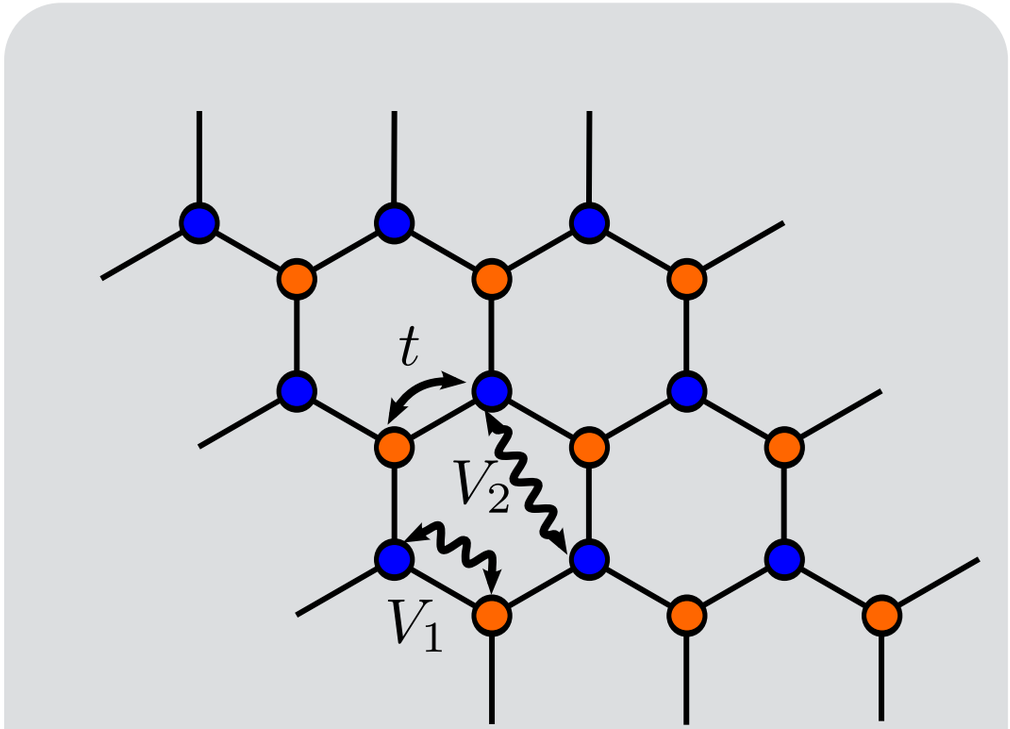
Fractional Chern Insulators

- Numerical evidence for a first order metal to Fractional Chern Insulator phase transition

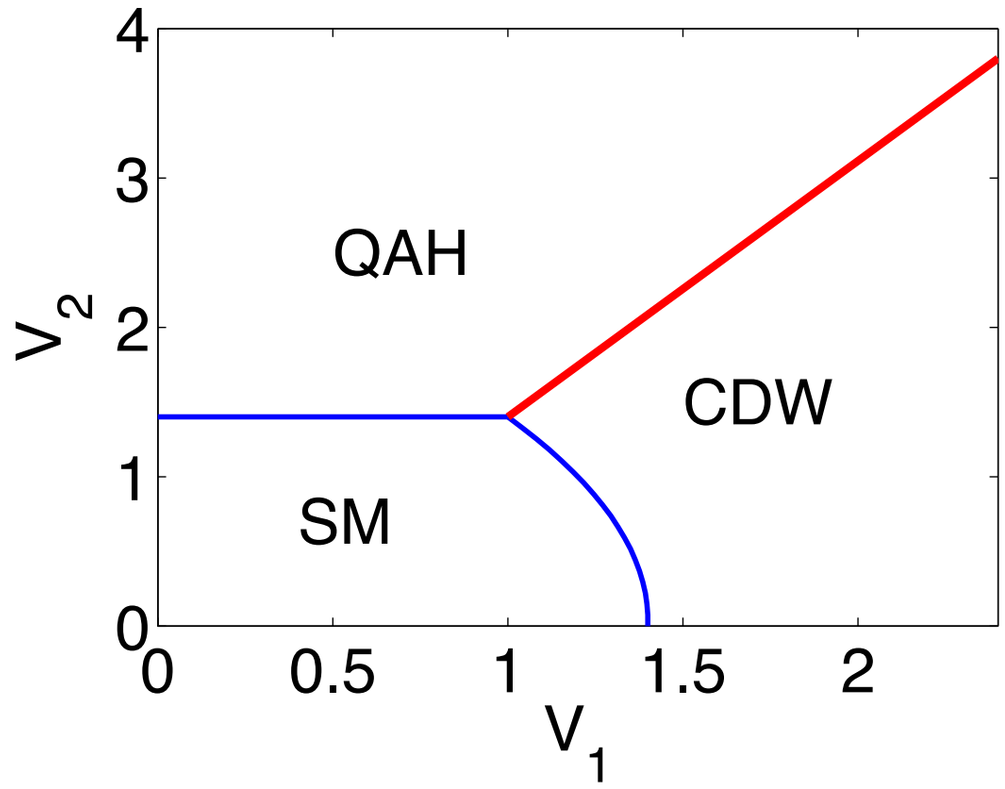


A. Grushin, J. Motruk, M. P. Zaletel, FP, Phys. Rev. B 91, 035136 (2015).

Interaction driven Chern insulators



[Raghu '08 et al.]



C. Weeks and M. Franz '10
A. G. Grushin et al., '13
N. A. García-Martínez et. al '13
B. Valenzuela, and E. V. Castro '13
M. Daghofer and M. Hohenadler '14
T. Durić, N. Chancellor, and I. F. Herbut '14
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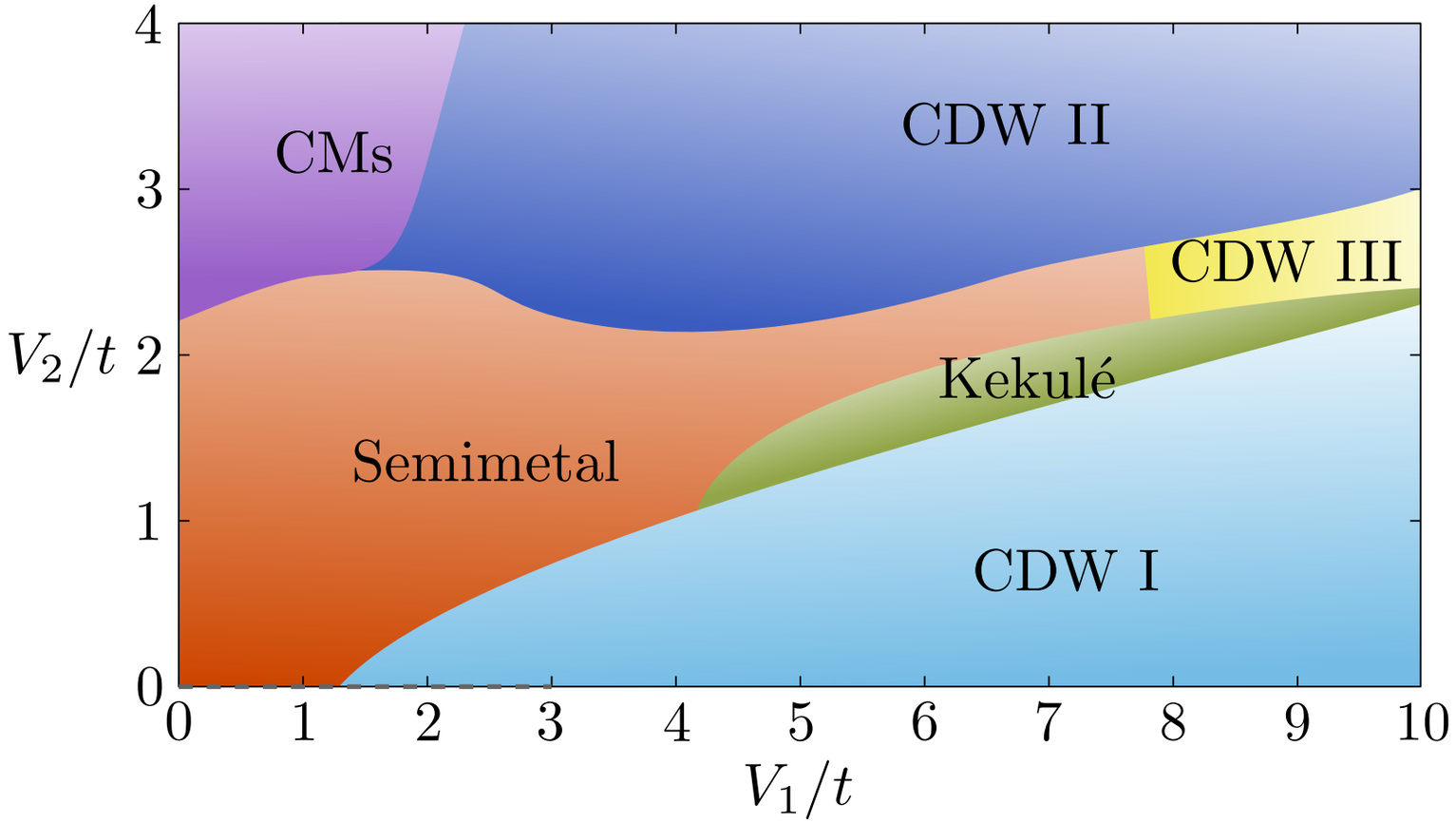
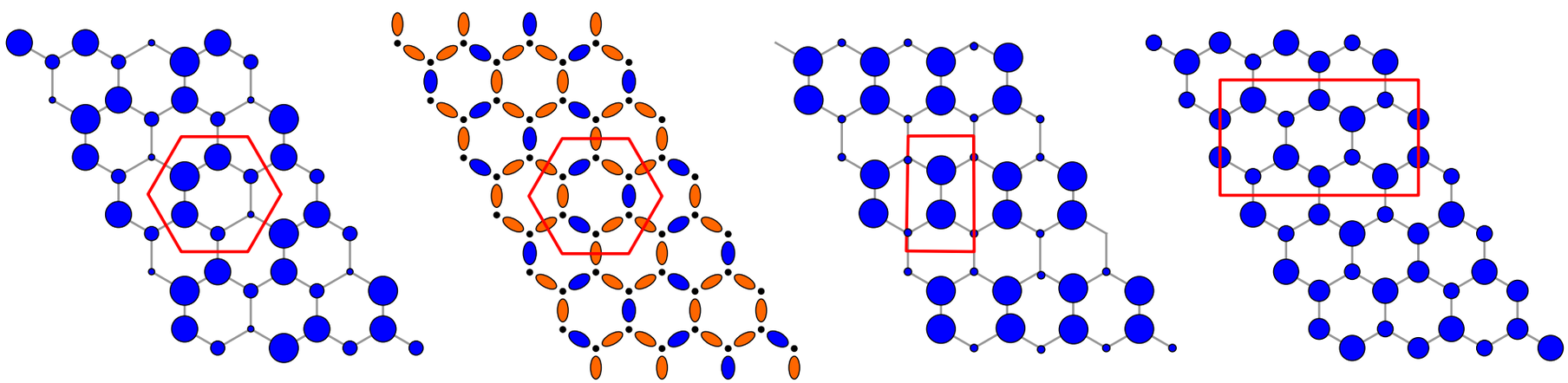
Interaction driven Chern insulators



Adolfo Grushin, MPIPKS
Johannes Motruk, MPIPKS
Fernando de Juan, Berkeley



J. Motruk, A. Grushin, F de Juan, FP (in preparation)

Summary

- Characterization of intrinsic topologically ordered systems
  - Numerical evidence for the existence of Fibonacci anyons in $\nu = 12/5$ FQH
  - Stability of an FCI phase in the Haldane model
  - Absence of an interaction driven Chern insulator on the honeycomb lattice model

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Thank You!